

Syllogisms

Aristotle as a Criterion of Validity

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Introduction

The research is based not on any new understanding and/or terminology of any sort; it rather depends on concepts and basic understandings in syllogistic theory, mainly as Aristotle formulated it, such as "necessity", "possibility" and "logical consequence". These concepts, particularly within the domain of syllogism, are related and interconnected with each other not only for Aristotle but also for modern logicians; the way modern logicians understand these concepts is not different from that of Aristotle.

Necessity, for Aristotle as well as for modern logicians, is a negation of possibility, i.e., if necessity is what cannot be otherwise, possibility is what can be. Thus, necessity and possibility are opposites and cannot hold together. Both, necessity and possibility have an interrelationship with logical consequences; logical consequence means that the truth of the conclusion follows of necessity from the truth of the premisses, for Aristotle as for Modern logic. Thus, both concepts are related to the question of truth.

Relying on this understanding, I find several forms of arguments, which are considered until this very day as valid, to be problematic.

Necessity versus Possibility

The question of certainty is a major concern in philosophy, namely, "to secure an adequate classification of truths, of the propositions expressing truths, and of the ways of coming to know truths" (Grayling 1982, p. 43) since the beginning of Greek philosophical thinking. For it is insufficient to produce truths; one must know how and when he produces truths.

In *Prior Analytics*, Aristotle solved the problem of the certainty of truth by discussing "the two central problems of logic as formal epistemology: how to show that a given conclusion follows from given premisses that formally imply it

and how to show that a given conclusion does not follow from given premisses that do not formally imply it" (Corcoran 2003, p. 262). Implying the conclusion, as a necessary consequence, is the way by which Aristotle secured, formally, an adequate way of acquiring certainty. Acquiring certainty by making the necessity of the conclusion evident is actually ascertaining and ensuring the truth of the conclusion of any argument (Lennox 2006, p. 3) if the premisses are true.

Necessitating the consequence necessitates formulating certain relationships between the three terms of the premisses, i.e., it necessitates specifying, through formulating, the relationships between the terms of the premisses as an act of regulation. Regulating the relationships between the terms is in itself a regulation of the relationship between the premisses and the conclusion. To be more accurate, regulating the relationships between the terms is in itself a regulation of the conclusion, and thus it is of necessity:

"If a deduction is possible the terms must be related as described, and if they are so related there will be a deduction." (Prior Analytics 26a 15)

The possibility of having more than one relationship between the two extreme terms of any given two premisses means that the relationship between them is variable. The varying of the relationship between the two extreme terms shows that what should define and determine the relationship between them, as a necessary consequence, is absent. In such a case, the premisses would not contain what is necessary for the relationship between their three terms to make the necessity evident. This lack has to do with the classification of valid/invalid forms of arguments: not being able to assert, as necessity, the relationship between the two extreme terms of two given premisses contradicts the mere possibility of syllogism. It rather indicates uncertainty and probability. The classical understandings of probability "assigns probabilities in the absence of any evidence, or in the presence of symmetrical balanced evidence. The guiding idea is that in such circumstances, probability is shared equally among all the possible outcomes. (Hájek 2003, pp. 4-5) That, of course, reminds us of Hume and his definition of chance as "nothing real in itself, and properly speaking, is merely the negation of cause, its influence on the mind is contrary to that of causation; and 'tis essential to it, to leave the imagination perfectly indifferent." (Hume 1976, p. 11). Leaving "the imagination perfectly indifferent" is the

problem that Aristotle wanted to solve by his formal deductive method on the bases of causality and necessity.

Thus, having different relationships between the two extreme terms of two given premisses means leaving the imagination perfectly indifferent, that is, there is no deductive conclusion. Consequently; these pairs of premisses would not be adequate as means by which we understand *why the thing is*, i.e., by which we prove *what the thing is*.

Multiple Conclusions

The validity of the forms of arguments that include a particular premise, which Aristotle examined in his work and which are considered until this very day as valid, is what I intend to investigate and which I find to be problematic. These forms of arguments are:

All A is B
Some A is C
∴ Some C is B

All A is B
Some C is not B
∴ Some C is not A

| | |
|-------------------|-------------------|
| All A is not B | All A is not B |
| Some A is C | Some C is B |
| ∴ Some C is not B | ∴ Some C is not A |

Relying on the modern understanding of validity, these forms of arguments are all valid, for each of them has a conclusion that its negation cannot possibly be true. On the other hand, the premisses of these forms of arguments do not prevent other possible relationships between their two extreme terms. In the first form argument mentioned above, for example, the two relationships “Some C is B” and “All C is B” are equally possible given the two premisses “All A is B” and “Some A is C”. If we analyze the two premisses, so as to uncover the information that each of them contains and the information they together carry, we see the possible relationships there could be between the two extreme terms C and B of this form.

The premisses are:

All A is B

Some A is C

The universal positive premise should necessarily include enough information about the subject term of the premise, term A, but it does not contain much information about the predicate, term B, for it is not distributed. In other words, we do not know if 'Some B is not A' is true, and if it is true, we do not know whether this object is necessarily or accidentally B. The particular premise does not include much information about the term C: we only know that there is at least one C that has property A.

The particular premise refers to the relationship C has with A, Some C is A, and this entails, together with the information included in the first premise, that there cannot be a negative universal relationship between the two extreme terms: a relationship with A is actually a relationship with B. Thus, if the premisses are true, the negation of the conclusion 'Some C is B' cannot possibly be true.

However, this information does not entail the kind of relationship C and B have: the two premisses together inform us that at least one C has both properties, A and B. This does not entail that C would have property B if and only if it has property A, and there is no reason to think that. Let me say it differently: since we do not know whether 'Some B is not A' is true or not, we have no reason to think that the particular relationship between C and A leads to particular relationship between C and B, as the only possible relationship they can have. This thinking could be necessarily true if and only if 'Some B is not A' is necessarily not true: if 'Some B is not A' is true and this object (B which is not A) has property C, then 'All C is B' is possible. As an example for the universal case, take the two premisses 'All humans are mortal' and 'Some humans are males'. The relationship here, between the two extreme terms, is not 'Some males are mortal', but 'All males are mortal'.

The relationship between human and males did not lead to the only relationship males can have with mortal; it did not lead to particular relationship between the terms, but to universal. This means that the two premisses are not the

explanation of the conclusion, i.e., of the particular relationship C can have with B.

Thus, one can assert that the two relationships 'Some C is B' and 'All C is B' are equally possible in this form of argument.

However, it would appear that the particular and the universal propositions, according to Aristotle, cannot be both true:

“What is most universal is further away, and the particulars are nearest; and these are opposite to each other.” (Posterior Analytics, 2 72a1 5)

For Aristotle, "All C is B" means that if it is C then it is necessarily B, that is, it is impossible to be C and not B, given that the universal is "whatever belongs to something both of every case and in itself and as such. It is evident, therefore, that whatever is universal belongs *from necessity* to its object." (Posterior Analytics, 73b1 25, *emphasis mine*) The relationship between the terms of the universal premisses is a relationship of what is impossible.

On the other hand, 'Some C is B', means it is possible to be C and not B, given that the two particular premisses, the positive and the negative, can both be true. This means that the relationship between the terms of the particular premisses is not a relationship where something belongs "to something both of every case and in itself", and "what does not belong because of itself is *accidental*." (Posterior Analytics, 73b1 10, *emphasis mine*) The relationship between the terms of the particular premisses is a relationship of what is possible, or as an alternative, of what is not impossible.

Having equally possible relationships, one of possibility and one of impossibility, between the two extreme terms of any form of argument means having two statements as two explanations of *what the thing is*; one says that C is necessarily B, and the other says it is possibly B. This, in turn, means that the middle term does not fulfill its role as an explanatory term (Posterior Analytics, 90a1), that is, it does not explain the relationship between the two extreme terms of the premisses, and consequently, the two essential questions: "*what the thing is?*" and "*why the thing is?*" cannot be answered. Thus, this form of argument should have been considered as invalid. .

One should make the clarification that the question here is not about the possibility of inferring the truth of the particular premise from that of the universal, for in this case, the relationship between the terms of the particular, as of the universal, is of

necessity and thus, can be both true. Within the domain of syllogism, where we infer the conclusion from two premisses, we have to take the particular and the universal as their definition, i.e., as opposite to each other, and as such they cannot be both true. For to answer the question “*what the thing is?*”, as a necessary consequence of two premisses, is to explain whether the relationship between the two extreme terms of any two given premisses is of possibility or of impossibility, as well as to ascertain it as a necessary consequence, i.e., as a logical consequence.

Now, taking the particular and the universal as opposites, both premisses, the particular positive and the universal negative, are false where the universal positive is true. The truth of the positive universal conclusion, in the first form of argument of my concern, means that C is necessarily B, not possibly B nor necessarily not B. Thus, it is clear that the principle of contradiction does not hold within the domain of syllogism, nor in any domain where we take into consideration the definition of the premisses, i.e., where we take the particular and the universal as opposites and cannot be both true. Consequently; proving that the negation of ‘Some C is B’ of the two premisses ‘All A is B’ and ‘Some A is C’, cannot possibly be true is not a proof of validity, for they both can be untrue and the universal positive is the true one. Let me explain it differently, following Aristotle, the particular positive and the universal positive premisses are opposite to each other, and hence, comparing with immediate inferences, they cannot be both true, but they can be both untrue. The truth of the positive particular indicates the untruth of the universal positive, but the untruth of the one does not indicate the truth of the other, for the universal negative can be the true premise. This is true for the particular negative, the universal negative and the universal positive premisses: the truth of the particular negative indicates the untruth of the universal positive as well as the untruth of the universal negative, but not *vice versa*. The truth of the particular negative premise means that C is possibly not B, not necessarily B nor necessarily not B.

Thus, comparing with immediate inferences where the particular positive and the universal negative cannot be both untrue, within syllogism each of the three premisses stands by itself, declaring different statement, and thus, none of them can be true with the other, but each two can be untrue where the third is true. ‘C is necessarily B’ cannot possibly be true with ‘C is possibly B’ or with ‘C is necessarily not B’. The truth of necessarily B indicates both, the untruth of necessarily not B and the untruth of possibly B.

Consequently, using the principle of contradiction where each premise stand by itself would mislead us.

Now, regardless of the differences between the universal and the particular premisses, i.e., suppose there are no principled differences between the particular and the universal premisses; following the way Aristotle investigated and classified the forms of arguments; having two possible relationships between the two extreme terms of one form of argument indicates invalidity. The clarification of the relevance of having two possible relationships (one particular and one universal) between the two extreme terms of one form of argument, should in itself exhibit the invalidity of the form.

For Aristotle, who associated “every syllogistic pair which forms a syllogism with just one type of conclusion” (Duerlinger 1968, p. 497), and who defined the conclusion as a “proposition that follows from certain premisses with necessity” (Patzig 1959, pp. 188 – 189); proving necessity is proving the impossibility of being otherwise than it is and *vice versa*, or, as Lear puts it, since “a syllogistic consequence follows of necessity from the premisses, to establish sterility one must prove a certain possibility” (Lear 1980, p. 54).

Investigating the possibility of having more than one relationship between the two extreme terms is the way by which Aristotle investigated every single form of argument in the *Prior Analytics*. The investigation Aristotle held concerned the arrangements of the terms so as to see whether the relationship between the two extreme terms alternate or not:

“Since we know when a deduction can be formed and how its terms must be related, it is clear when refutation will be possible and when impossible. A refutation is possible whether everything is conceded, or the answers alternate.” (*Prior Analytics*, Book II, 20)

A proof of possibility is a proof of sterility whether we consider Aristotle’s syllogisms a body of implications as Lukasiewicz claims (Lukasiewicz 1963, p. 21), or a logical system as Corcoran (Corcoran 1974) and Smiley (Smiley 1973) believe. For “Aristotle’s view of the world demands a distinction between what *cannot* be otherwise and what *can* be otherwise” (Rini 2011, p. 40). Here one should assert that, according to Aristotle, the universal syllogisms allow more than one necessary conclusion because their conclusions can be converted. For

the same reason the particular affirmative syllogisms allow more than one necessary conclusion:

“Since some deductions are universal, others particular, all the universal deductions give more than one result, and of particular deductions the affirmative yield more than one, and the negative only the stated conclusion. For all propositions are convertible save only the particular negative; and the conclusion states one thing about another.” (Prior Analytics, Book II, 1 53a1 5-10)

Regardless of the problematic embedded in the method of conversion, the particular and the universal affirmative propositions are not and cannot be considered as a conversion of each other. That is, the particular and the universal affirmative conclusions are two different relationships we can have between the two extreme terms of the first form of argument of my concern. Consequently, whether we take the differences between the universal and the particular positive premisses or not, this form of argument should have been considered as invalid.

Here I would claim that through the very long history of logic, the effort logicians invested, was not to investigate, but to validate the forms accepted by Aristotle. One can make an extreme statement and say that the acceptance of Aristotle itself was the criterion of validity.

This approach, I claim, explains why logicians consider some forms of arguments, in particular these of my concern, as valid until this very moment.

The methods of testing validity

The examination of validity is the way through which categorical syllogism is ascertained (Shaw 1997, p. 37). Questioning and investigating the forms of arguments is a way by which logicians either assert validity or assert invalidity. In either case, the investigation of validity should be held on the basis of the understanding that syllogism is a method of generating true conclusions from true premisses by necessity. In this context, I should add that the differences in logic generated after Frege, regarding necessity and validity within the domain of syllogism, did not make any difference in principle. Rather; one principle was added to the Aristotelian principles of inference, and the role of this added

principle (which forbids inferring a particular conclusion from two universal premisses), together with the other principles, is to ascertain the necessity of the inference and of the truth of the conclusion given the truth of the premisses.

The methods of testing the validity of categorical forms of syllogistic arguments, such as Venn Diagrams, have been playing an important role in distinguishing between what can be recognized as valid forms of arguments and what can be dismissed as invalid. A certain method of assessment is predominantly followed, however, simply due to being able to adhere to previously accepted results. Reviewing the different systems of diagrams (Carroll 1896, Venn 1881, Euler 1768) which attempted to establish the validity of the forms of arguments shows that the acceptance or rejection of the system of diagrams was conditioned by its ability to validate, through graphical representations, the forms of arguments that were traditionally accepted as valid. While Euler Diagrams were rejected due to their failure to validate the forms of arguments that include a particular premise (such as 'All A is not B', 'Some C is B', \therefore 'Some C is not A', which is one of the forms of my concern¹), Venn Diagrams were accepted precisely for their ability to validate all forms of arguments that were traditionally accepted as valid (Shin & Limon 2003, Diagrams). The same as to Carroll diagrams, "for on what grounds does one believe that the rules for filling in a Lewis Carroll diagrams will distinguish the valid from the invalid inferences? Is it not because the rules harmonize with our pre-existing beliefs about which inferences are valid?" (Lear 1980, 59 - 60)

Thus, the accepted forms of arguments were thus used as themselves a criterion of validity. To use the traditionally accepted forms of arguments to validate the method of testing their validity means that the validation is circular. Thus, instead of examining the forms of arguments, these forms themselves have been used for validating and explaining the way the methods work, including the method Lukasiewicz proposed.

The essence of Aristotle's definition of deduction corresponds to the modern understanding of logical consequence: for Aristotle, a "deduction is speech in which, certain things having been supposed, something different from those supposed results of necessity because of their being so" (*Prior Analytics* 24b18-20), and according to the modern understanding of logical consequence, A

¹ The possible relationships between the two extreme terms of this form of argument are 'Some C is not A' and 'All C is not A'.

results of necessity from P and Q if it would be impossible for A to be false when P and Q are true.

Conclusion of necessity is a conclusion that cannot be false given two true premisses. Modern logicians falsify the conclusion "Some C is B" of the two premisses "All A is B" and "Some A is C" by giving an example where "All A is B", "Some A is C" and "All C is not B" are all together true. A necessarily true conclusion, within the domain of modern logic, is a conclusion whose contradiction cannot be true. The same as to Aristotle; he proved the necessity of the conclusion hypothetically:

“For all who effect an argument *per impossibile* de duce what is false, and prove the original conclusion hypothetically when something impossible results from the assumption of its contradictory.” (Prior Analytics 41a1 23-25)

Thus, the conclusion that its contradiction cannot be true is a conclusion of necessity. Consequently, the two formulations below are synonymous:

1. An inference is valid just in case the truth of the premisses guarantees the truth of the conclusion.
2. An inference is valid just in the case when the premisses are true, and the contradiction of the conclusion cannot be true.

Using the two above formulations as synonymous, means giving the meaning “cannot be contradicted” to “necessarily true”, and consequently, both would indicate validity:

“Deductions which lead to impossible conclusions are similar to probative deductions; they also are formed by means of the consequents and antecedents of the terms in question. In both cases the same inquiry is involved. For what is proved probatively may also deduced *per impossibility* by means of the same terms; and what is proved *per impossible* may also proved probatively.” (Prior Analytics 45a1 23-27)

One can trace a certain inconsistency here, for if according to Aristotle “some deductions are universal, others particular, all the universal deductions give more than one result, and of particular deductions the affirmative yield more than one, and the negative only the stated conclusion. For all propositions are

convertible save only the particular negative”, (Prior Analytics, 53a1 5-10) then, though a refutation of deduction is possible when “everything is conceded, or the answers alternate (one, I mean, being affirmative, the other negative)”, (Prior Analytic 66b1 6-7) proving *impossibility* should not be taken as a proof of validity. The impossibility of ‘All C is not B’ when ‘All A is B’ and ‘Some A is C’ are true, is not but a proof that the contradiction of ‘Some C is B’ is not possible. ‘All C is not B’ is one alternation that cannot possibly be. The impossibility of ‘All C is not B’ is not a proof of the impossibility of other alternations of ‘Some C is B’, i.e., it is not a proof that not everything is conceded, or that the answer does not alternate at all.

For this reason, Patzig agreeing with Lukasiewicz, considered the proof *per impossibility* fallacious:

“Aristotle has, to sum up, been puzzled by the logical fact that a conclusion correctly inferred from false premisses is neither by necessity true nor false. He has, to overcome his difficulty in understanding this simple fact, tried to device a proof of it. But he failed: the main premiss of his proof (what I called his “principle” is false, and the attempted demonstration of this principle by *reductio ad absurdum* is fallacious.” (1959, p. 192).

The explanation Lukasiewicz gives for his objection is that the indirect proof should start from the negation of the mood, “and not from the negation of its conclusion and the negation should lead to an unconditionally false statement, and not to a proposition that is admitted to be false under certain conditions.” (Lukasiewicz 1957, p. 56) Thus, he proposes an alternative proof for the forms that were traditionally accepted as valid, using the following propositional laws: $((p \wedge q) \rightarrow r) \rightarrow ((p \wedge \neg r) \neg q)$.

The question of syllogism is not whether one conclusion is possible or not, but one of ascertaining necessity, i.e., what is important is not what cannot be true, but what cannot be but true. Consequently, taking the conclusion that its negation cannot possibly be true as necessarily true, would not solve the problem Lukasiewicz wanted to solve. For, as was explained above, this is not a proof that the conclusion does not alternate; it is rather a proof that one among three possible relationships is unconditionally false, and as such, it is fallacious. Proving that one statement is unconditionally false, as an alternative of Aristotle proof, is nothing but another attempt to validate the forms accepted by Aristotle.

However, the interesting in Lukasiewicz attempt is that his proof falls within the understandings of Aristotle. That is, Lukasiewicz not only accepts the forms accepted by Aristotle, but relies on the understandings Aristotle relied on as well, and thus, he commits the same mistake.

Bibliography

1. Aristotle, *The Complete work of Aristotle*, V. 1, Edit by Barnes, J. Princeton University Press, Guildford, Surrey, 1984
2. Carroll, L. *Symbolic Logic*, Dover, 1896
3. Copi, A. *Introduction to Logic*, Macmillan Publishing CO, 1972
4. Corcoran, J. "*Aristotle's Prior Analytics and Boole's Laws of Thought*", *History and Philosophy of Logic*, 24 (2003), 261 – 288
5. Corcoran, J. "*Aristotelian Syllogisms: Valid Arguments or True Universal Conditional?*", *Mined*, Vol. 83, No. 330 (1974), pp. 278 - 281
6. Duerlinger, J. "*Aristotle's Conception of Syllogism*", *Mined*, Vol. 77, No. 103 (1968), pp. 480 - 499
7. Euler, L. *Letters a'une Princesse d'Allemagne*. St. Petersburg; l'Academie Imperiale des Ssiences 1768
8. Euler, L. *Letters a'une Princesse d'Allemagne sur divers sujets de physique duction et des notes par E'mile Saissest*. Paris; Charpentier 1859
9. Grayling, A. C. *An Introduction to Philosophical Logic*, Harvester Press, 1982
10. Hájek, A. "*Interpretations of Probability*", *Stanford University Encyclopedia, 2007*
11. Hume, D. *Enquiries, Concerning the Human Understanding and Concerning the Principles of Morals*, Second Edition, L. A. Selby, M. A., Oxford, 1976
12. Lear, J. *Aristotle And Logical Theory*, Cambridge, Cambridge University Press, 1980
13. Lennox, J, "*Aristotle Biology*", *Stanford University Encyclopedia*, 2006
14. Lukasiewicz, J. *Aristotle's Syllogistic From The Standpoint of Modern Formal Logic*, Clarendon Press, Oxford, 1963
15. Patzig, G. "*Aristotle and Syllogisms from False Premisses*", *Mined*, Vol. 68, No. 270 (1959), pp. 186 - 192
16. Rini, A. *Aristotle's Modal Proofs – Prior Analytics A8 – 22 in Predicate Logic*, Springer, 2011
17. Shaw, P. *Logic and Its Limits*, Oxford University Press, 1997
18. Shin, S., Limon, O. "*Diagrams*", *Stanford University Encyclopedia*, 2003

19. Smiley, T. J. "*What is Syllogisms?*", journal of Philosophical Logic, Vol. 2, No. 1 (1973), pp. 136 - 154
20. Venn, J. *Symbolic Logic*, London, Macmillan, 1881